EXCEPTIONAL SEQUENCES AND DERIVED CATEGORIES

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Let k be an algebraically closed field of characteristic 0. We denote by $\mathcal{C}$ the abelian k-category which has enough projectives (or enough injectives), and by $\mathfrak{D}^b(\mathcal{C})$ the bounded derived category of $\mathcal{C}$.

A complex $\mathrm{E}^\bullet \in \mathfrak{D}^b(\mathcal{C})$ is called *exceptional* if $\mathrm{RHom}(\mathrm{E}^\bullet, \mathrm{E}^\bullet) \cong k$, and a sequence $\epsilon = (\cdots, \mathrm{E}^\bullet{}_i, \mathrm{E}^\bullet{}_{i+1}, \cdots)$ of exceptional complexes is called an *exceptional sequence* if $\mathrm{RHom}(\mathrm{E}^\bullet{}_i, \mathrm{E}^\bullet{}_j) = 0$ for all $i > j$.

Let $\mathcal{C}$ be a category $\operatorname{mod} A$ of finitely generated modules of a hereditary k-algebra A, or a category $\mathrm{coh}(\mathbf{X})$ of coherent sheaves of a weighted projective line $\mathbf{X}$ over k. In this case, for any exceptional sequence ϵ, the length of ϵ is at most the rank n of Grothendieck group of $\mathcal{C}$. An exceptional sequence ϵ is called *complete* if the length of ϵ is equal to n. It was shown by W. Crawley-Boevey (in the case of $\mathcal{C} = \operatorname{mod} A$) and by H. Meltzer (in the case of $\mathcal{C} = \mathrm{coh}(\mathbf{X})$) that the braid group B_n on n strings acts *transitively* on the set of complete exceptional sequences. It was also shown that the complete exceptional sequence ϵ generates $\mathfrak{D}^b(\mathcal{C})$ as a triangulated category.

In this lecture, I will talk about the case of $\mathcal{C} = \operatorname{mod} R$ (where R is a 1-dimensional $\mathbb{N}$-graded Gorenstein ring of finite Cohen-Macaulay representation type).

References

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