AB rings and AB modules

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Let (R, m, k) be a commutative Noetherian local ring. We denote by $\mathrm{mod}R$ the category of finitely generated R-modules.

Definition 1 For non-zero R-modules M and N, we define $\mathrm{P}_R(M, N)$ as follows:

$$\mathrm{P}_R(M, N) = \sup\{\ n \mid \mathrm{Ext}^n_R(M, N) \neq 0\ \}$$

Definition 2 We say that R is an *AB-ring* if the following condition holds:

$$\sup\{\ \mathrm{P}_R(M, N) \mid \mathrm{P}_R(M, N) < \infty\ (M, N \in \mathrm{mod}R)\ \} < \infty$$

An AB-ring was introduced by C.Huneke and D.A.Jorgrnsen in [2]. They consider the following question.

Question Are Gorenstein rings AB-rings ?

The answer is No. D.A.Jorgensen and L.M.Şega showed that there exist Gorenstein rings which are not AB-rings. On the other hand, it is known that there exist AB-rings which are not Gorenstein rings.

In this lecture, I talk about properties of modules over AB-rings.

References

[1] T.Araya and Y.Yoshino, Remarks on a depth formula, a grade inequality and a conjecture of Auslander, *Comm. Algebra* **26** (1998), no. 11, 3793-3806.

[2] C.Huneke and D.A.Jorgensen, Symmetry in the vanishing of Ext over Gorenstein rings, *Math. Scand.* **93** (2003), 161–184.

[3] D.A.Jorgensen and L.M.Şega Nonvanishing cohomology and classes of Gorenstein rings, *Adv. Math.* **188** (2004), 470–490