Hook Formulas for a Generalized Young Diagram

Kento NAKADA
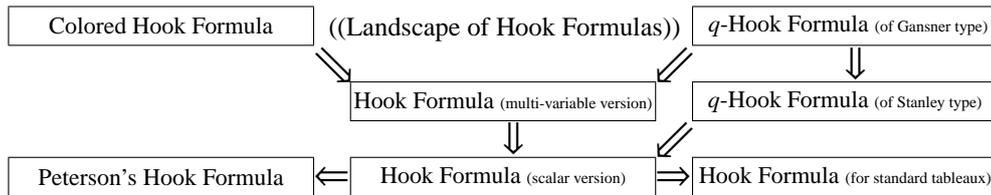
Let λ be a partition of d, and χ_λ the corresponding irreducible character of the symmtric group $\mathfrak{S}_d$. As is well-known, the degree $\chi_\lambda(1)$ of χ_λ is given by the hook formula:

$$(1) \qquad \chi_\lambda(1) = \frac{d!}{\prod_{v \in Y_\lambda} h_v},$$

where Y_λ is the Young (or Ferrers) diagram of shape λ, and h_v is the hooklength at a cell v of Y_λ. Since the left hand side of (1) is equal to the number $\#\mathrm{STab}(Y_\lambda)$ of standard tableaux of shape λ, the formula (1) can be rewritten as:

$$(2) \qquad \#\mathrm{STab}(Y_\lambda) = \frac{d!}{\prod_{v \in Y_\lambda} h_v}.$$

According to [1], around 1989, D. Peterson proved a formula which counts the number of reduced decompositions of minuscule [1][3] elements in Weyl group of Kac-Moody Lie algebra $\mathfrak{g}$ (Peterson's hook formula), and he noted that this formula gave a generalization of hook formula (2) for a Young diagram. However, these works (details and proofs) have been unpublished still now.

Colored Hook Formula	((Landscape of Hook Formulas))	q-Hook Formula (of Gansner type)
	Hook Formula (multi-variable version)	q-Hook Formula (of Stanley type)
Peterson's Hook Formula	Hook Formula (scalar version)	Hook Formula (for standard tableaux)

By introducing the concept "*finite pre-dominant* integral weight", we have succeeded in proving the *colored hook formula* [4] and the *q-hook formula* [5] for such an integral weight. Since we can regard a finite pre-dominant integral weight for a Kac-Moody Lie algebra of type A as a Young diagram, we can regard a finite pre-dominant integral weight as a generalized Young diagram.

By two steps specialization of these formula, we have secceeded in proving the Peterson's hook formula. Hence, the colored hook formula and q-hook formula are far more general than the Peterson's hook formula, and these are entirely new formulas. In fact, the colored hook formula is new even for a Young diagram.

REFERENCES

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Graduate School of Information Science and Technology, Osaka University, Toyonaka, Osaka 560-0043, Japan. e-mail address : u300883f@ecs.cmc.osaka-u.ac.jp