Torsion points of abelian varieties with values in infinite extensions over a p-adic field.

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Let K be a finite extension field of the p-adic number field $\mathbb{Q}_p$ and fix an algebraic closure $\bar{K}$ of K. Let A be an abelian variety over K. If $L \subset \bar{K}$ is a finite extension over K, it is well-known that the torsion part of the L-rational points $A(L)$ is finite. On the other hand, in general, we do not know whether the torsion part of $A(L)$ is finite or infinite if $L \subset \bar{K}$ is an infinite algebraic extension over K. We are interested in understanding whether the torsion part of $A(L)$ is finite or infinite. In a global case (that is, the case where K is a number field), such an argument has been studied and there are lots of results. But in a local case, it seems not to be disucussed more precise than a global case. In this connection H. Imai proved that the torsion points of $A(K(\mu_{p^\infty}))$ is finite for an abelian variety A over K which has good reduction. In this talk first we consider a generalization of a special case of Imai's theorem: Let A be an abelian variety over K which has potentially ordinary good reduction. (1) Let L be an algebraic extension of K such that residue field k_L is potentially prime-to-p extension of k. Then $A(L)[p^\infty]$ is finite. (2) Let L be an algebraic extension of K whose residue field is finite. Then the torsion part of $A(L)$ is finite.

Next we try to generalize Imai's theorem in another way. Let B be a semiabelian variety over K. We denote by $K_{B,p}$ the field generated by the p-power torsion of B. If B is the multiplicative group $\mathbb{G}_m$ over K, $K_{B,p} = K_{\mathbb{G}_m,p}$ is equal to the cyclotomic field $K(\mu_{p^\infty})$. Hence Imai's theorem is a result with respect to the torsion part of $A(K_{\mathbb{G}_m,p})$. From such a point of view, we raise the question whether the torsion part of $A(K_{B,p})$ is finite or infinite. For example, by applying our first result and an argument of Serre, we can show that the torsion part of $A(K_{B,p})$ is finite if the abelian variety A has ordinary good reduction over K and B is an elliptic curve with supersingular good reduction or multiplicative reduction over K.

It seems not to be known an (in-)finiteness property of the torsion part of $A(K_{B,p})$ in many case. We consider such a finiteness problem with restricting to the p-part of $A(K_{B,p})$ (for any prime $\ell \neq p$, we can prove easily that the ℓ-part of $A(K_{B,p})$ is finite). In this talk we give some finiteness results on the case where A is an elliptic curve. For example, in the case where A and B is elliptic curves, we prove that $A(K_{B,p})[p^\infty]$ is infinite in many case if A and B has ordinary good reduction, however, $A(K_{B,p})[p^\infty]$ is finite in many case if A has supersingular good reduction.